1 -There are three types of fuzzy inference systems (controllers), explain with examples the operation of these types.

These fuzzy inferences (models/controllers) are:

(1)Mamdani fuzzy inference

The Mamdani-type fuzzy inference was first proposed as an attempt to control a steam engine and boiler using a set of linguistic control rules obtained from an experienced human operator.

EX: Consider a two input single output Mamdani fuzzy model, each input x1, x2 and output y has two MFs: {A1, A2}, {B1, B2} and {C1, C2}, respectively. If we consider two rules R1 and R2, these rules are:

R1 : IF x1 is A1 AND x2 is B1 THEN y is C1

R2 : IF x1 is A2 AND x2 is B2 THEN y is C2

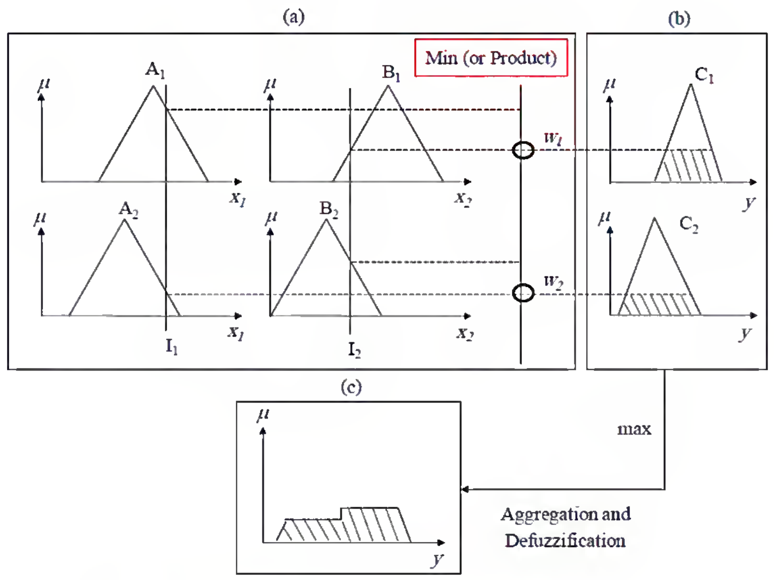


Fig. 1 Two-input single-output Mamdani fuzzy model

In the Mamdani -type fuzzy model shown in Fig.1, two measured crisp values I1 and I2 are used for the inputs x1 and x2, respectively.

• Fig.1-a shows the fuzzification and inferencing using the minimum rule (AND operator in the IF-Part can be represented using Min. or product rule) for computing the firing strengthsw1 andw2 for the premise terms of the rules.

w1 = min(μA1 ,μB1) , w2 = min(μA2 ,μB2)

w1 = μA1 . μB1 , w2 = μA2 . μB2

• w1 and w2 are stand for μPremise1 and μPremise2 as we used before, these weights represent the strengths for the firing rules.

The inferred output of each rule is the truncated membership functions chosen from the minimum firing strength as shown in Fig.1-b. The truncated membership functions for

each rule are aggregated as shown in Fig.1-c and any of the following defuzzification methods (like Center of gravity (COG)) is carried out to convert a fuzzy set to a crisp value, these methods are:
• Center of Gravity (COG) method
• Weighted average method
• Mean-max membership method

(2)Sugeno fuzzy inference
The Sugeno fuzzy inference, also known as the TSK fuzzy model, was proposed by Takagi, Sugeno and Kang in 1985.
• The output of each rule of the fuzzy IF-THEN rules (consequent or then part) is a linear function which is a combination of input variables plus a constant term.
• **EX:** Consider a two- input single-output TSK fuzzy model, each input x1 and x2 has two membership functions (MFs) {A1 , A2} and {B1 , B2}, if we consider two rules R1 and R2 with consequent functions {y1 , y2}.These rules are:
R1 : IF x1 is A1 AND x2 is B1 THEN y1 = p1 x1 + q1 x2 + r1
R2 : IF x1 is A2 AND x2 is B2 THEN y2 = p2 x1 + q2 x2 + r2

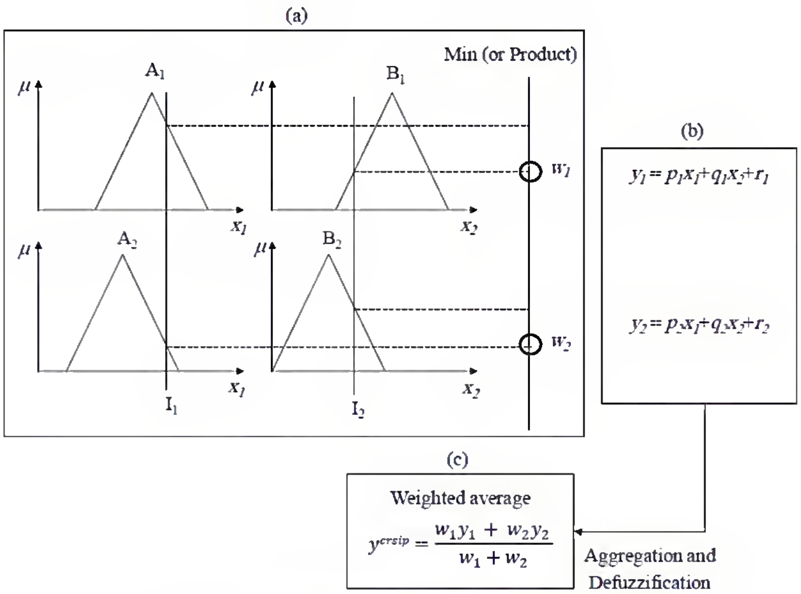


Fig. 2 Two-input single-output TSK fuzzy model

In the Sugeno-type fuzzy model shown in Fig.2, two measured crisp values I1 and I2 are used for the inputs x1 and x2, respectively.
• Fig. 2-a shows the fuzzification and inferencing using the minimum or product rule for computing the firing strengthsw1 andw2 for the premise term of the rules.
The firing strength is calculated using the minimum or product rule as:

w1 = min(μA1 ,μB1) , w2 = min(μA2 ,μB2)

or

w1 = μA1 . μB1 , w2 = μA2 . μB2

• w1 and w2 are stand for μPremise1 and μPremise2 as we used before, these weights represent the strengths for the firing rules.
Once the parameters {p1, q1, r1, p2, q2, r2} are known, the consequent y1 and y2 are calculated for each rule using a first-order polynomial as shown in fig.2-b. The overall output y is obtained via the weighted average of the crisp outputs y1 and y2.
The weighted average defuzzification method is computed by:

$$y\text{crisp} = (w1y1 + w2y2)/(w1 + w2)$$

• Fig.2-c illustrates the aggregation and final defuzzified value for the TSK model.

(3)Tsukamoto fuzzy inference

In the Tsukamoto fuzzy inference, the consequent of each fuzzy if then rule is represented by a monotonic MF (function between ordered sets that preserves the given order).

EX: To illustrate the Tsukamoto-type mechanism, consider a two input single output Tsukamoto fuzzy model, each input x1 and x2 has two membership functions (MFs) {A1,A2} and {B1,B2}, if we consider two rules R1 and R2 with consequent monotonic functions {C1,C2}.These rules are:
R1: IF x1is A1 AND x2is B1THEN y is C1
R2: IF x1is A2AND x2is B2THEN y is C2

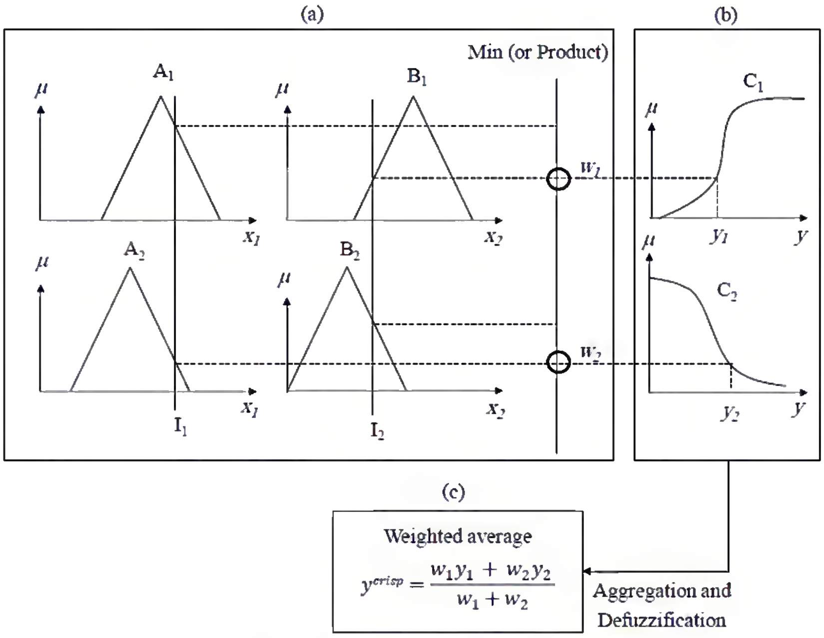


Fig. 3 Two-input single-output Tsukamoto fuzzy model

In the Tsukamoto-type fuzzy model shown in Fig.3, two measured crisp values I1 and I2 are used for the inputs x1 and x2, respectively.

• Fig.3-a shows the fuzzification and inferencing using the minimum or product rule for computing the firing strengths w1 and w2 for the premise term of the rules. The firing strength is calculated using the minimum or product rule.

• The consequent y1 and y2 represent the defuzzified outputs (one value for each rule) and are determined as shown in fig.3-b.

• The overall output y is obtained via the weighted average of the crisp outputs y1 and y2. The overall output y is obtained via the weighted average of the crisp outputs y1 and y2. The weighted average defuzzification method is computed by:

$$y\text{crisp} = (w1y1 + w2y2)/(w1 + w2)$$

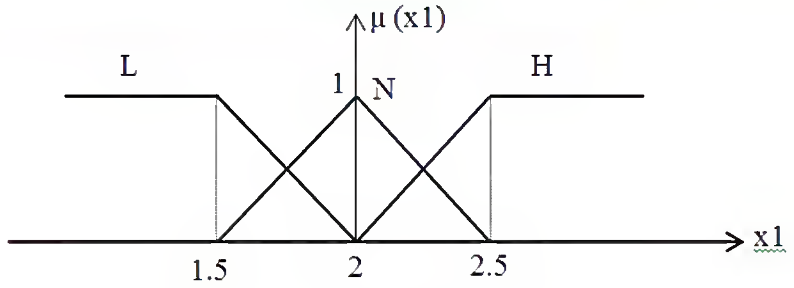
• Fig.3-c illustrates the defuzzified value for the Tsukamoto model.

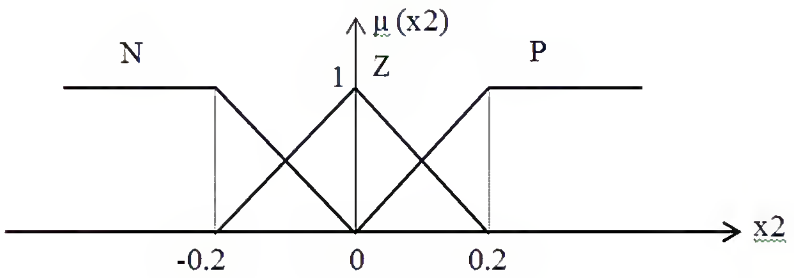
2- What are the main differences between Mamdani, TSK, and Tsukamoto fuzzy controllers?

Mamdani type fuzzy inference gives an output that is a fuzzy set. Sugeno-type inference gives an output that is either constant or a linear (weighted) mathematical expression. Tsukamoto fuzzy inference, the consequent of each fuzzy if then rule is represented by a monotonic MF (function between ordered sets that preserves the given order).

e.g Mamdani: If A is X1, and B is X2, then C is X3. (X1, X2, X3 are fuzzy sets).
Sugeno: If A is X1 and B is X2 then C = ax1 + bx2 + c (linear expression) (a, b, and c are constants)
Tuskamoto: R1: IF x1is A1 AND x2is B1THEN y is C1

3- A TSK fuzzy controller is designed for a level control process, with two inputs: x1 (represent the level) and x2 (represent the rate of change in level). The output of the controller is u (represent the valve position). The MFs for the inputs x1 and x2 are in the following graph:





The rules of the controller are:

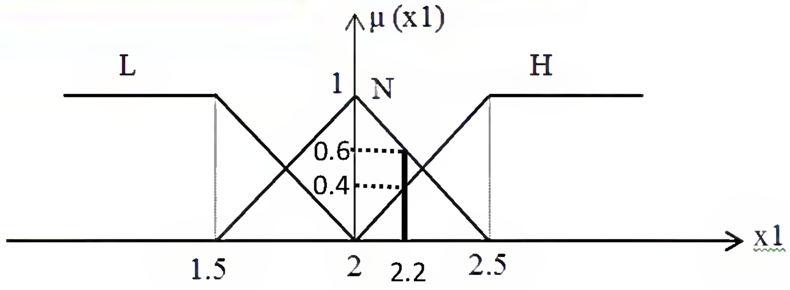
X1 \ X2	N	Z	P
L	Y1	Y1	Y1
N	Y2	Y2	Y3
H	Y4	Y4	Y4

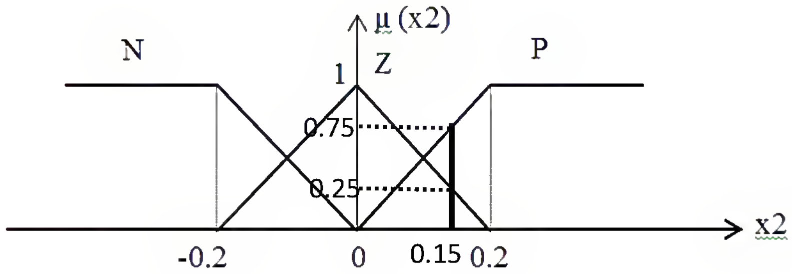
Where: y1 = 4 x1 – 0.25 x2 + 0.05

y2 = x1 -0.1 x2
y3 = 0.5 x1 – 0.1 x2
y4 = 0.2 x1 – x2

Find the controller crisp output (ucrisp) when:

- **X1=2.2 and X2=0.15**





1- Fuzzification

x1 is N with μ*N* (x1 = 2.2) = 0.6
x1 is H with μ*H* (x1 = 2.2) = 0.4
x2 is Z with μ*Z* (x2 = 0.15) = 0.25
x2 is P with μ*P* (x2 = 0.15) = 0.75

2- The Fired Rules:

R1: IF x1 is N AND x2 is Z THEN y2 = x1 -0.1 x2
R2: IF x1 is N AND x2 is P THEN y3 =0.5 x1 – 0.1 x2
R3: IF x1 is H AND x2 is Z THEN y4 = 0.2 x1 – x2
R4: IF x1 is H AND x2 is P THEN y4 = 0.2 x1 – x2

Where at x1 **= 2.2 ,** x2 **= 0.15 :**

y2 = 2.2 – (0.1*0.15)= 2.185
y3 = (0.5*2.2) - (0.1*0.15)= 1.085
y4 = (0.2*2.2) – 0.15= 0.29

2- The Fired Rules:

R1: IF x1 is N AND x2 is Z THEN y2 = 2.185
R2: IF x1 is N AND x2 is P THEN y3 =1.085
R3: IF x1 is H AND x2 is Z THEN y4 = 0.29
R4: IF x1 is H AND x2 is P THEN y4 = 0.29

3-The strength of the fired rules:

R1: w1= μpremise1=min {μ*N*(x1),μ*Z*(x2)}=min{*0.6,0.25}*= 0.25
R2: w2= μpremise2 =min {μ*N*(x1),μ*P*(x2)}=min{*0.6,0.75}*= 0.6
R3: w3= μpremise3=min{ μ*H*(x1),μ*Z*(x2)}=min{*0.4,0.25}*= 0.25

R4: w4= μpremise4 =min {μ*H*(x1),μ*P*(x2)}=min{*0.4,0.75}*= 0.4

4-Aggregation and Defuzzification:
R1: y1= 2.185 with w1= 0.25
R2: y2= 1.085 with w2= 0.6
R3: y3= 0.29 with w3= 0.25
R4: y4= 0.29 with w4= 0.4
Using weighted average method
ucrisp= (*w*1*y*1+*w*2*y*2+*w*3*y*3+*w*4*y*4) / (*w*1+*w*2+*w*3+*w*4)
ucrisp= (0.25*2.185+0.6*1.085+0.25*0.29+0.4*0.29) / (0.25+0.6+0.25+0.4)=0.924

- **x1 = 2.15 and x2 = -0.15**

1- Fuzzification
x1 is N with μ*N* (x1 = 2.15) = 0.7
x1 is H with μ*H* (x1 = 2.15) = 0.3
x2 is Z with μ*Z* (x2 = - 0.15) = 0.25
x2 is N with μ*N* (x2 = - 0.15) = 0.75

2- The Fired Rules:
R1: IF x1 is N AND x2 is Z THEN y2 = x1 -0.1 x2
R2: IF x1 is N AND x2 is N THEN y2 = x1 -0.1 x2
R3: IF x1 is H AND x2 is Z THEN y4 = 0.2 x1 – x2
R4: IF x1 is H AND x2 is N THEN y4 = 0.2 x1 – x2

Where at x1 **= 2.15 ,** x2 **= - 0.15 :**
y2 = 2.15 – (0.1*- 0.15)= 2.165
y4 = (0.2*2.15) + 0.15= 0.58

2- The Fired Rules:
R1: IF x1 is N AND x2 is Z THEN y2 = 2.165
R2: IF x1 is N AND x2 is N THEN y2 =2.165
R3: IF x1 is H AND x2 is Z THEN y4 = 0.58
R4: IF x1 is H AND x2 is N THEN y4 = 0.58

3-The strength of the fired rules:
R1: w1= μpremise1=min {μ*N*(x1),μ*Z*(x2)}=min{*0.7,0.25}*= 0.25
R2: w2= μpremise2 =min {μ*N*(x1),μ*N*(x2)}=min{*0.7,0.75}*= 0.7
R3: w3= μpremise3=min{ μ*H*(x1),μ*Z*(x2)}=min{*0.3,0.25}*= 0.25
R4: w4= μpremise4 =min {μ*H*(x1),μ*N*(x2)}=min{*0.3,0.75}*= 0.3

4-Aggregation and Defuzzification:
R1: y1= 2.165 with w1= 0.25
R2: y2= 2.165 with w2= 0.7
R3: y3= 0.58 with w3= 0.25
R4: y4= 0.58 with w4= 0.3
Using weighted average method
ucrisp= (*w*1*y*1+*w*2*y*2+*w*3*y*3+*w*4*y*4) / (*w*1+*w*2+*w*3+*w*4)

ucrisp= (0.25*2.165+0.7*2.165+0.25*0.58+0.3*0.58) / (0.25+0.7+0.25+0.3)=1.58

- **x1 = 1.75 and x2 = -0.1**

1- Fuzzification
x1 is N with μN (x1 =1.75) = 0.5
x1 is L with μL (x1 = 1.75) = 0.5
x2 is Z with μZ (x2 = - 0.1) = 0.5
x2 is N with μN (x2 = - 0.1) = 0.5

2- The Fired Rules:
R1: IF x1 is N AND x2 is Z THEN y2 = x1 -0.1 x2
R2: IF x1 is N AND x2 is N THEN y2 = x1 -0.1 x2
R3: IF x1 is H AND x2 is Z THEN y1 = 4 x1 – 0.25 x2 + 0.05
R4: IF x1 is H AND x2 is N THEN y1 =4 x1 – 0.25 x2 + 0.05

Where at x1 **= 1.75 ,** x2 **= - 0.1 :**
y2 =1.75– (0.1*- 0.1)= 1.76
y1 = (4*1.75) – (0.25*- 0.1) +0.05=7.075

2- The Fired Rules:
R1: IF x1 is N AND x2 is Z THEN y2 = 1.76
R2: IF x1 is N AND x2 is N THEN y2 =1.76
R3: IF x1 is L AND x2 is Z THEN y1 = 7.075
R4: IF x1 is L AND x2 is N THEN y1 = 7.075

3-The strength of the fired rules:
R1: w1= µpremise1=min {µ*N*(x1),µ*Z*(x2)}=min{*0.5,0.5}*= 0.5
R2: w2= µpremise2 =min {µ*N*(x1),µ*N*(x2)}=min{*0.5,0.5}*= 0.5
R3: w3= µpremise3=min{ µ*L*(x1),µ*Z*(x2)}=min{*0.5,0.5}*= 0.5
R4: w4= µpremise4 =min {µ*L*(x1),µ*N*(x2)}=min{*0.5,0.5}*= 0.5

4-Aggregation and Defuzzification:
R1: y1= 1.76 with w1= 0.5
R2: y2=1.76 with w2= 0.5
R3: y3=7.075 with w3= 0.5
R4: y4=7.075 with w4= 0.5
Using weighted average method
ucrisp= $(w1y1+w2y2+w3y3+w4y4)$ / $(w1+w2+w3+w4)$
ucrisp= (0.5*1.76+0.5*1.76+0.5*7.075+0.5*7.075) / (0.5+0.5+0.5+0.5)=4.4175

4- What is meant by ANFIS? To what type of controller it belongs?

- The ANFIS is a fuzzy system which is modelled in the form of the artificial neural network (ANN) so that a learning algorithm can be used to train the system.
- ANFIS was introduced by **Jang in 1993**

- ANFIS stands for Adaptive Neuro Fuzzy Inference System or, Adaptive Network-based Fuzzy Inference System.
- TSK fuzzy inference system is simple in computation and easy to be combined with optimizing and self-adapting methods, so that ANFIS based on TSK fuzzy inference system is the most type that is commonly used.
- It belongs to **Computational Intelligence Controllers**

5-What are the parameters to be optimized in ANFIS?

The parameters to be optimized in ANFIS are the premise (antecedent or IF part) parameters which describe the shape of the MFs, and the consequent (conclusion or THEN part) parameters which describe the overall output of the system

6- Explain with examples the types of methods that used to optimize ANFIS parameters?

- Various methods have been previously proposed to optimize (train) ANFIS parameters. These methods can be divided into two types:

1- derivative-based methods   2- derivative-free methods

- **Derivative-based methods** include backpropagation (BP), least squares estimate (LSE), and hybrid learning (HL). HL is a combination of LSE and BP.

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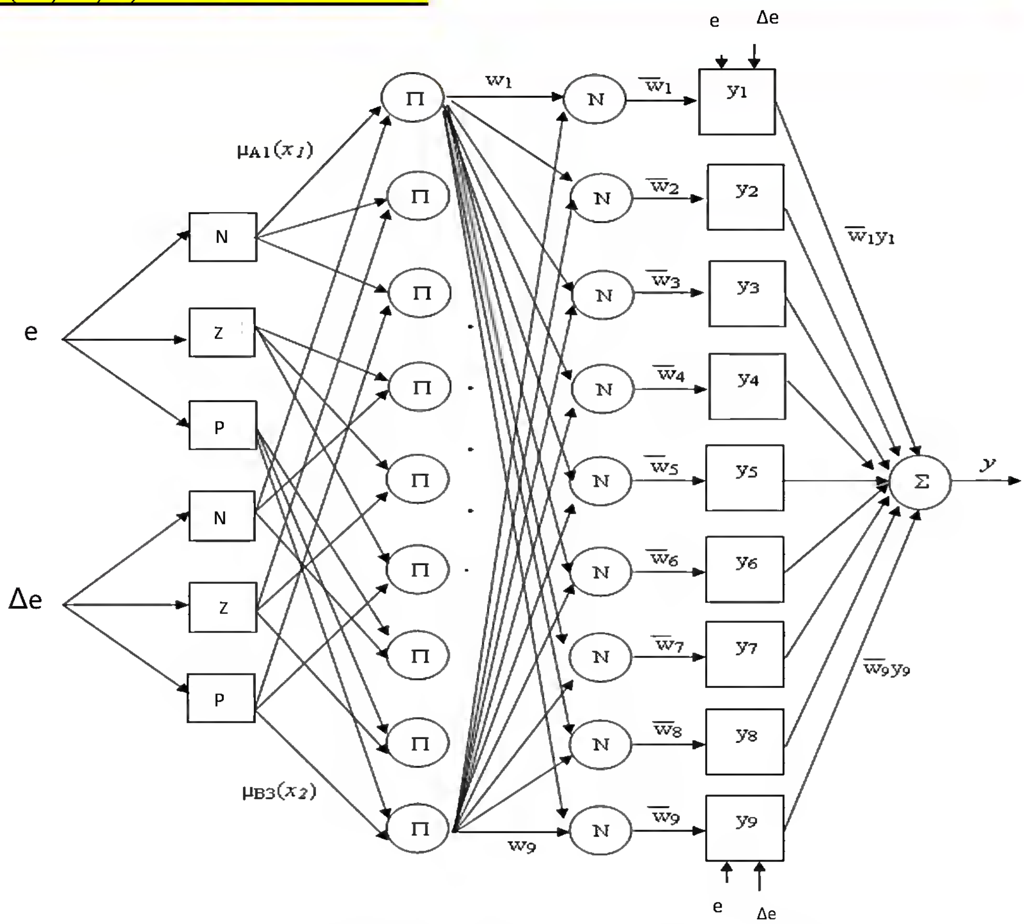
- **Derivative-free methods** include other evolutionary computation algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), differential evolution (DE), shuffled frog leaping Algorithm (SFLA), artificial bee colony algorithm (ABC).

7-Consider the ANFIS model has 2 inputs, the first input has 3 triangular MFs and the second input has 5 triangular MFs. How many rules and parameters of ANFIS to be optimized?

- The total No. of rules = 3 x 5 = 15 rule
- The parameters to be optimized (or tuned) in ANFIS are the premise (IF part) parameters and the consequent (THEN part) parameters.
- No. of premise parameters = No. of control parameters for MF x total No. of MFs
  (The triangular MF has 3 control parameters.)
  No. of premise parameters =3 x (3+5) = 24 parameter
- No. of consequent parameters = (No. of inputs + 1) x No. of rules
  = (2+1) x (3x 5) = 45 parameter

- The total No. of ANFIS parameters =
  No. of premise parameters + No. of consequent parameters
  = 24+45=69 parameter

8- Draw the structure of the ANFIS model that has two input (e , Δe) when three MFs (N , Z , P) are used for the two.



The ANFIS structure with two inputs, nine rules and one output